

GCE

Further Mathematics A

Y542/01: Statistics

Advanced GCE

Mark Scheme for Autumn 2021

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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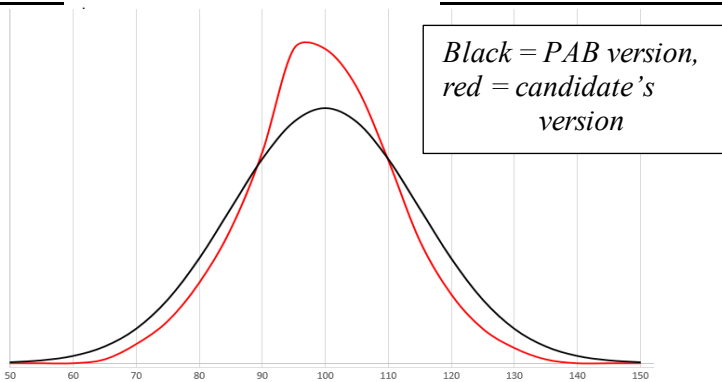
Annotations and abbreviations

Annotation in RM assessor	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Question			Answer	Marks	AO	Guidance	
1	(a)		$y = 52.7 + 0.251x$	B1* B1* depB1 [3]	1.1 1.1 1.1	a in range [0.250, 0.251] b correct to 3 SF Completely correct including letters SC: Correct formulae used for a and b : M1(A1)A1	
1	(b)		This quantity is minimised to find best-fit line	B1 [1]	2.4	Need “minimised” or “this is its minimum value” OE	
1	(c)		$y' = 11.5 + 0.139x$ $[y' = \frac{5}{9} \times (\text{their } a - 32) + \frac{5}{9} \times \text{their } b]$	M1 A1ft [2]	1.1 1.1	Apply inverse formula at least once All correct, any letters, ft on their y	
2			$E(D) = 2 \times 0.1 + 4 \times 0.3 + 6 \times 0.2$ $= 2.6$ $E(D^2) = 2^2 \times 0.1 + 4^2 \times 0.3 + 6^2 \times 0.2$ [= 12.4] $\text{Var}(D) = 12.4 - 2.6^2$ $= 5.64$ $\text{Var}(3D + 4) = 9 \times \text{Var}(D)$ $= 50.76$	M1 A1 M1 M1 A1 M1 A1 [7]	2.1 1.1 1.1 1.1 1.1 3.1a 3.4	NB: a is not needed by this method Or $\Sigma(x - \mu)^2 p(x)$ $\Sigma p^2 d$ oe gets max M1A1M0M1M1 Allow even if their $\text{Var}(D) < 0$ SC: $\Sigma(x - \mu)^2 p(x)$: M1A1, $a = 0.4$ M1 M1(use this formula), A1M1A1	Or change 0, 2, 4, 6 to 4, 10, 16, 22 and find a
3	(a)	(i)	$P(X \geq 5) - P(X \geq 11) = 0.7^4 - 0.7^{10}$ $= 0.212$	M1 A1 [2]	3.1b 3.4	Allow 1 term wrong at either end awrt 0.212	Or $pq^4 + \dots + pq^9$
		(ii)	$0.7^{n-1} < \frac{1}{3}$, or $0.103 > 0.1 > 0.072$ $n_{\min} = 5$	M1 A1 [2]	2.1 1.1	Solve $0.3 \times 0.7^{n-1} = 0.1$ or < 0.1 , allow inequality error 5 only SC: 5 without sufficient justification: B1	
3	(b)		$\frac{1-p}{p^2} = 42 \Rightarrow 42p^2 + p - 1 = 0$ $p = \frac{1}{7}$ Explicitly reject $p = -\frac{1}{6}$ $E(X) = 7$	M1 A1 A1 A1 A1	3.1a 1.1 2.2a 2.3 2.1	Equate correct variance formula to 42 Correct simplified quadratic equation SC: if $-\frac{1}{7}$ and $\frac{1}{6}$, allow A1 for explicitly rejecting $-\frac{1}{7}$	

Question			Answer	Marks	AO	Guidance
				[5]		
4	(a)	(i)	$\hat{\mu} = \bar{x} = 16.8$	B1 [1]	1.1	Or exact equivalent
		(ii)	$\frac{48398}{160} - 16.8^2 \quad [= 20.2475]$ $\times \frac{160}{159}$ $= 20.3748\dots$	M1 M1 A1 [3]	1.1 1.1 1.1	If single formula used, full marks if correct; M0M1 if wrong but divisor 159 seen anywhere Awrt 20.4, www
4	(b)		$\bar{x} \pm z\sqrt{\sigma^2 / 160}$ $z = 2.576$ (15.88, 17.72)	M1 A1 A1 [3]	3.3 1.1 3.4	Any z from Φ^{-1} , 160 needed, allow $\sqrt{}$ errors Or better, e.g. 2.575829 Both, 4 sf required by question, www (NB: $\sigma^2 = 20.2475$ gives same end-points to 4 SF but this gets M1A1A0)
4	(c)	(i)	Not needed in (a) as $E(X)$ and $\text{Var}(X)$ are independent of the distribution	B1 [1]	2.4	Mention at least one of $E(X)$ and $\text{Var}(X)$ explicitly, or “not relevant to $\bar{X}$ ”
		(ii)	Needed in (b) as parent distribution not stated to be normal	B1 [1]	2.4	Must make it clear that two distributions are involved. “ n is large” etc: B0

Question		Answer	Marks	AO	Guidance
5	(a)	The value of Pearson's pmcc would be changed by (most) such changes.	B1	2.5	Explain effect on Pearson, <i>or</i> not known bivariate normal <i>or</i> not testing for <i>linear</i> correlation
		The value of Spearman's r_s would not be changed as the ranks remain unchanged.	B1 [2]	2.5	Explain why no effect on Spearman (not “not <i>likely to be</i> affected”, or “not <i>much</i> affected” or “association not correlation”)
5	(b)	H_0 : no association between ranks of numbers of items	B1	1.1	Don't insist on “population” here, but allow use of ρ_s in both, even if no explanation (not just r_s). Context needed, but don't worry about 1- or 2-tailed here
		H_1 : (positive) association between ranks			
		Ranks 1 2 3 4 5 6 7 8 9 4 1 3 2 8 5 9 7 6	M1	1.1	
		$\Sigma d^2 = 38$	A1	1.1	
		$r_s = 1 - \frac{6\Sigma d^2}{9(9^2 - 1)}$	M1	1.2	Compare TS ($-1 \leq TS \leq 1$) with 0.7, independent ft on TS provided correct formula used, or on CV 0.600 In context, not too positive. FT on TS only SC: 0.600 (2-tailed): B0 M1A0
		$= 0.683$	A1	1.1	
6	(a)	H_0 : Data consistent with $N(100, 15^2)$	B1	1.1	Allow: “follows $N(100, 15^2)$ ” or “can be modelled by”. Parameters not needed. No other alternatives seen!
		H_1 : Data not consistent with $N(100, 15^2)$	[1]		
6	(b)	$P(100 \leq X < 110) = 0.2475$ BC			Probability needs to be seen Sufficient working to justify AG , needs 123.754 at least
		Expected frequency = $500 \times 0.2475 [= 123.754]$	B1	3.4	
		$\frac{(129 - 123.754)^2}{123.754} [= 0.222..., \mathbf{AG}]$	M1	2.1	
			A1 [3]	2.2a	

Question			Answer	Marks	AO	Guidance	
6	(c)		$\Sigma X^2 = 10.5$ $\chi^2(4) = 9.488$ and $10.5 > 9.488$ Reject H_0 . Significant evidence that data is not consistent with $N(100, 15^2)$.	B1 B1 M1ft A1ft [4]	1.1 1.1 1.1 2.2b	Like-with-like comparison needed FT on TS or CV here. Needn't be stated if next line right FT on TS (but not CV) if method correct. Wrong CV, e.g. 5.991: B1B0M1A0. No ft on H_0/H_1	
6	(d)	(i)	E.g. Too few in $X \geq 110$ or in $X \leq 80$, or too many in others, or data truncated, etc	B1 [1]	3.5b	Any relevant point, needn't refer to values of X^2 "Divide into 5 minute groups": B1. "Data discrete": B0. "The variance" (uncalculated): B0	
		(ii)		B1 B1 [2]	3.3 3.5c	Deal with aspect identified in (i) Basically correct, areas roughly same <i>Examples:</i> Uses "data discrete" in (i) More below 100, so translate to left More above 110 so translate to right Divide into 5-minute groups Variance changed, areas not equal Data truncated but worse truncation shown	 B0 B2 B2 B0 B1 B0
7	(a)		H_0 : Two samples are from identical populations H_1 : Two samples are from populations with different median ratings. $R_m = 1 + 2 + 3 + 4 + 5 + 9 + 10 + 11$ (= 45) $W = 45$ $8(8 + 8 + 1) - R_m = 91$ $W_{crit} = 49$ Reject H_0 . Significant evidence that there is a difference in median ratings/opinions have changed	B2 M1 A1 B1 B1 M1ft A1ft [8]	1.1 1.1 1.1 1.1 1.1 1.1 2.2b	If no reference to "populations", maximum B1 Allow H_0 : "identical population medians", H_1 : "not identical populations" or "not identical pop medians" "Pupils' opinions have not changed", etc: B2 If omitted, can still get all other marks FT on TS (< 68) or CV FT on TS only. Allow "increased" SC: Sign or paired-sample test, max B2 (hypotheses)	
7	(b)		Eliminate the difference between individual pupils' opinions	B1 [1]	3.5b	"Minimises the difference in tastes" B1 (BOD) Scores arbitrary: B1 (etc). Not "more powerful test".	
7	(c)		A paired-sample signed-rank test would have been used	B1	3.5c	Must mention "paired sample" oe – not just "Wilcoxon"	

Question			Answer	Marks	AO	Guidance
				[1]		signed rank”
7	(d)		0.025×12870 $= 322$	M1 A1 [2]	3.1a 3.2a	$0.05 \times 12870 = 643.5$ M1 321 or 322 or 643 (from 1-tail), must be integer

Question			Answer	Marks	AO	Guidance
8	(a)		$f(x) = \frac{1}{2}$	B1	3.3	Stated or implied, e.g. on diagram
			$\int_0^2 \frac{1}{2} a \cos(ax) dx = 0.3$	M1	3.1a	$\int f(x) a \cos ax \, dx$ & equated to 0.3
			$\left[\frac{1}{2} \sin(ax) \right]_0^2$	B1	1.1	Correct indefinite integral
			$\frac{1}{2} \sin(2a) = 0.3$	M1	2.1	Correct limits, solve
			$a = 0.32175\dots$	A1 [5]	1.1	Answer, a.r.t. 0.322 (ignore other answers)
8	(b)		$F(y) = \frac{1}{2} y \quad [0 \leq y \leq 2]$	M1	3.1a	Use their $f(y)$ to obtain CDF
				A1	1.1	Correct $F(y)$ (range need not be stated explicitly)
			$P(Y^2 \leq m) = P(0 < Y \leq \sqrt{m})$	M1	2.1	Find CDF of Y^2 , allow m^2 instead of $\sqrt{m}$, or $\pm\sqrt{m}$, here
			$= F(\sqrt{m}) \quad [= \frac{1}{2}\sqrt{m}]$	A1	1.1	Use $F(y)$ correctly
			$\frac{1}{2}\sqrt{P_{60}} = 0.6$	M1	1.1	Equate to 0.6 and solve, need $\sqrt{m}$ here
			$P_{60} = 1.44$	A1 [6]	2.2a	1.44 or exact equivalent

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